DMT Signals with Low Peak-to-Average Power Ratio

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Abstract

This contribution proposes a new family of methods to reduce the Peak-to-Average power Ratio (PAR) in Discrete Multi-Tone (DMT) and Orthogonal Frequency Division Multiplexing (OFDM) systems. This method avoids an use of extra Inverse Fast Fourier Transformations (IFFTs) as was done in some previously published techniques, but instead is based on transformations in the time domain of the original sequence available at the output of the IFFT processor. The improved statistics of peak power in the optimized transmit signal are demonstrated by simulation results.

1. Introduction

Multi-carrier modulation has recently being used for many applications such as high-speed voice band modems and digital subscriber line systems (xDSL). These systems use DMT over wire media and OFDM for wireless communication.

The amplitude distribution of a DMT signal with random input data is approximately Gaussian for large number of carriers. Therefore the DMT signal will occasionally present very high peaks. These peaks require a high dynamic range of the ADC (analog to digital converter) and analog front end in absence of any clipping or peak reduction technique. This would result in inefficient amplifiers (with excessive power dissipation) and expensive transceivers.

Recently, there has been a variety of proposed methods to reduce PAR [2]-[6], but none of these methods is able to achieve simultaneously a large reduction in PAR with low complexity, without performing an extra IFFT calculations, without loss in data rate, without increase in the transmitter average power and without decrease in the minimum distance or equivalently without loss in the noise margin

This paper proposes a new technique based on [1], which can achieve all these goals.

2. The Proposed Method to reduce PAR

The basic characteristic of DMT is that the data are modulated on N QAM subcarriers X_k $(k = 0,1,\ldots,N-1)$ each of equal bandwidth. This frequency multiplexing can easily be implemented by using an Inverse Fast Fourier Transform (IFFT) in the modulator that transforms the subcarrier vector $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^t$, with $X_i = X^*_{N-i+2}$ for $i = 2,\ldots,N/2$ and t denotes the transpose and $*$ the complex conjugate, into the time domain. This results in the discrete time representation $\mathbf{x} = [x_0, x_1, \ldots, x_{N-1}]^t$ with

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi}{N}kn} \qquad n = 0,1,\ldots,N-1 \tag{1}$$

The sequence $\mathbf{x}$ is real and is often referred to as a DMT symbol. It's well known that for large N, the output time vector distribution can be well approximated by a Gaussian distribution and has a relatively large PAR defined as:

$$PAR(\mathbf{x}) = \frac{|x_{\max}|^2}{\sigma_x^{\,2}} \tag{2}$$

where $x_{\max}$ and σ_x are respectively the maximum value of x_n and the root-mean square value of $\mathbf{x}$ over all possible symbols.

The real sequence given by Eq. (1) can also be obtained by using a complex IFFT of the modulated carriers which are organized in the form of the following complex sequence $Y(n)$:

$$Y(n) = \begin{cases} \dfrac{X(n)}{2} & \text{for } n = 0, \dfrac{N}{2} \\ X(n) & \text{for } 0 < n < \dfrac{N}{2} \\ 0 & \text{for } \dfrac{N}{2} < n \le N-1 \end{cases} \tag{3}$$

Herein, $X(n)$ is a complex value representing the n'th modulated carrier of the multi-carrier symbol to be transmitted, $\frac{N}{2}$ is the number of carriers, and n is an integer index. The IFFT of the complex sequence $Y(n)$ is another complex sequence $y(n)$ denoted as:

$$y(n)=a(n)+jb(n) \qquad (4)$$

Herein, $a(n)$ and $b(n)$ are two real sequences of length N and $j=\sqrt{-1}$. The output $y(n)$ of the IFFT is a complex time domain sequence of length N whose real part $a(n)$ will be selected for transmission by the real part selector REAL. It is further remarked that:

$$x(n)=2a(n)=2\operatorname{Re}[y(n)] \qquad (5)$$

if $x(n)$ represents the real IFFT of the multi-carrier symbol $X(n)$, i.e. the IFFT of the sub-carrier complex conjugate vector built up from $X(n)$.

When the real part of a sample $a(n)$ of the complex time domain multi-carrier symbol $y(n)$ exceeds a certain threshold value, the time domain multi-carrier symbol is subjected to a transformation. The threshold condition is verified by the observation device OBS which thereupon controls the transformation unit TRANSF to apply the transformation on $y(n)$. Observing the time domain multi-carrier symbol $y(n)$ and transforming it is executed iteratively. The proposed transformation unit TRANSF is able to apply seven different transformations on the complex time domain multi-carrier symbol $y(n)$. These seven transformations will be discussed further. If none of the seven transformations produces a transformed time domain multi-carrier symbol whose respective real part $t_1(n)$, $t_2(n)$, $t_3(n)$, $t_4(n)$, $t_5(n)$, $t_6(n)$ or $t_7(n)$ consist of samples with an amplitude below the threshold value, a clip event occurs. This may cause harmonics and out-of-band radiation on the communication line. Moreover, the clipping will result into an increase of the binary error rate.

In a first step, the transformation unit TRANSF decomposes the time domain multi-carrier symbol $y(n)$ in four partitions each containing information with respect to special subsets of carriers of the multi-carrier symbol $Y(n)$. Suppose thereto that the frequency domain multi-carrier symbol $Y(n)$ would be decomposed into two partitions, the first partition $\mathbf{Y_e}$ carrying the even numbered carriers and the second partition $\mathbf{Y_o}$ carrying the odd numbered carriers. These partitions can mathematically be expressed as follows:

$$Y_e(n)=\frac{1}{2}\left(1+e^{j\pi n}\right)\cdot Y(n) \quad \text{for } n=0,1,\ldots,N-1 \qquad (6)$$

and

$$Y_o(n)=\frac{1}{2}\left(1-e^{j\pi n}\right)\cdot Y(n) \quad \text{for } n=0,1,\ldots,N-1 \qquad (7)$$

whereby $Y(n)=Y_e(n)+Y_o(n)$.

Y_e and Y_o now can each be decomposed into two further partitions:

$$Y_0(n)=\frac{1}{2}\left(1+e^{j\pi\frac{n}{2}}\right)\cdot Y_e(n) \quad \text{for } n=0,1,\ldots,N-1 \qquad (8)$$

$$Y_2(n)=\frac{1}{2}\left(1-e^{j\pi\frac{n}{2}}\right)\cdot Y_e(n) \quad \text{for } n=0,1,\ldots,N-1 \qquad (9)$$

whereby $Y_e(n)=Y_0(n)+Y_2(n)$, and

$$Y_1(n)=\frac{1}{2}\left(1-e^{j\pi\frac{n}{2}}\right)\cdot Y_o(n) \quad \text{for } n=0,1,\ldots,N-1 \qquad (10)$$

$$Y_3(n)=\frac{1}{2}\left(1+e^{j\pi\frac{n}{2}}\right)\cdot Y_o(n) \quad \text{for } n=0,1,\ldots,N-1 \qquad (11)$$

whereby $Y_o(n)=Y_1(n)+Y_3(n)$.

Using the Fourier transform properties, it can be verified that the time domain sequences corresponding to the previous frequency domain sequences Y_e, Y_o, Y_0, Y_1, Y_2 and Y_3 are given by:

$$y_e(n)=a_e(n)+jb_e(n), \text{ with} \qquad (12)$$

$$a_e(n)=\frac{1}{2}\left(a(n)+a\left(n+\frac{N}{2}\right)\right) \quad \text{for } n=0,1,\ldots,N-1, \text{ and}$$

$$b_e(n)=\frac{1}{2}\left(b(n)+b\left(n+\frac{N}{2}\right)\right) \quad \text{for } n=0,1,\ldots,N-1;$$

and:

$$y_o(n)=a_o(n)+jb_o(n), \text{ with} \qquad (13)$$

$$a_o(n)=\frac{1}{2}\left(a(n)-a\left(n+\frac{N}{2}\right)\right) \quad \text{for } n=0,1,\ldots,N-1, \text{ and}$$

$$b_o(n)=\frac{1}{2}\left(b(n)-b\left(n+\frac{N}{2}\right)\right) \quad \text{for } n=0,1,\ldots,N-1;$$

and:

$$y_0(n)=a_0(n)+jb_0(n), \text{ with} \qquad (14)$$

$$a_0(n)=\frac{1}{2}\left(a_e(n)+a_e\left(n+\frac{N}{4}\right)\right) \quad \text{for } n=0,1,\ldots,N-1$$

$$b_0(n)=\frac{1}{2}\left(b_e(n)+b_e\left(n+\frac{N}{4}\right)\right) \text{ for } n=0,1,\dots,N-1$$

and:

$$y_1(n)=a_1(n)+jb_1(n), \text{ with} \tag{15}$$

$$a_1(n)=\frac{1}{2}\left(a_o(n)+b_o\left(n+\frac{N}{4}\right)\right) \text{ for } n=0,1,\dots,N-1$$

$$b_1(n)=\frac{1}{2}\left(b_o(n)-a_o\left(n+\frac{N}{4}\right)\right) \text{ for } n=0,1,\dots,N-1$$

and:

$$y_2(n)=a_2(n)+jb_2(n), \text{ with} \tag{16}$$

$$a_2(n)=\frac{1}{2}\left(a_e(n)-a_e\left(n+\frac{N}{4}\right)\right) \text{ for } n=0,1,\dots,N-1$$

$$b_2(n)=\frac{1}{2}\left(b_e(n)-b_e\left(n+\frac{N}{4}\right)\right) \text{ for } n=0,1,\dots,N-1$$

and:

$$y_3(n)=a_3(n)+jb_3(n), \text{ with} \tag{17}$$

$$a_3(n)=\frac{1}{2}\left(a_o(n)-b_o\left(n+\frac{N}{4}\right)\right) \text{ for } n=0,1,\dots,N-1$$

$$b_3(n)=\frac{1}{2}\left(b_o(n)+a_o\left(n+\frac{N}{4}\right)\right) \text{ for } n=0,1,\dots,N-1$$

Thus, the transformation unit TRANSF first decomposes the time domain multi-carrier symbol $y(n)$ in 4 partitions $y_0(n)$, $y_1(n)$, $y_2(n)$ and $y_3(n)$. Afterwards, the real and imaginary parts of these partitions will be used to constitute a transformed time domain multi-carrier symbol whose real part is denoted by $t_1(n)$, $t_2(n)$, $t_3(n)$, $t_4(n)$, $t_5(n)$, $t_6(n)$ or $t_7(n)$. More specifically, the carriers of a partition must be rotated over an angle which is an integer multiple of $\frac{2\pi}{N}$ radians, the integer multiple being proportional to the carrier index n, and will be rotated over a fixed angle equal for all carriers of that partition before the partitions are re-combined into the transformed time domain multi-carrier symbol with real parts $t_i(n), i=1,\dots,7$.

One of the transformed real time domain symbols $t_i(n), i=1,\dots,7$ is selected for transmission. It will be parallel to serial converted and transferred to the multi-carrier receiver. An indication of the applied transformation is modulated on the pilot carrier by the pilot tone modulator PILOT MOD and added to the multi-carrier symbol before transmission towards the receiver. Obviously, the applied transformation may be reported in another way to the multi-carrier receiver too. In the receiver, the transformed time domain multi-carrier symbol is again parallelised by the serial to parallel converter and converted into a transformed frequency domain multi-carrier symbol, respectively $T_1(n)$, $T_2(n)$, $T_3(n)$, $T_4(n)$, $T_5(n)$, $T_6(n)$ or $T_7(n)$, by the fast fourier transfomer FFT. The pilot carrier is monitored by the pilot tone demodulator PILOT DMOD, and the information demodulated from this pilot carrier is used to control the inverse transformation unit INV TRANSF so that the exact inverse transformation is applied to the transformed frequency domain multi-carrier symbol $T_i(n), i=1,\dots,7$. Since the inverse transformation is executed in the frequency domain, the transformations applied by the transformation unit TRANSF in the transmitter need to have a simple frequency domain equivalent to be usable in practice. When the transformation consists of a rotation proportional to the carrier index and a fixed rotation equal for all carriers of partition, this condition is obviously satisfied.

In the implementation of the presented technique, some specific phasor transformations are used for the inverse transformations. Such phasor transformations are defined by a phasor transformation vector $Z(n)$. In general this phasor transformation vector may be expressed as:

$$Z(n)=A\cdot e^{j\varphi_n} \quad \text{with } n=0,1,\dots,N-1 \tag{18}$$

Herein, A is a complex constant with unit magnitude in order to not decrease the minimum distance of the constellation, and φ_n is any multiple of $\frac{2\pi}{N}$ proportional to the index n, and can be enlarged by a fixed angle φ_0. In other words, φ_n can be written as:

$$\varphi_n=\varphi_0+n\frac{2\pi M}{N} \tag{19}$$

With M being an arbitrary integer and φ_0 being a constant angle. Generally, since the time domain multi-carrier symbol $y(n)$ is divided into four partitions $y_0(n)$, $y_1(n)$, $y_2(n)$ and $y_3(n)$, four transformations can be defined: $Z_1(n)$, $Z_2(n)$, $Z_3(n)$ and $Z_4(n)$. The multi-carrier symbol obtained at the output of the carrier transformer FFT then can be expressed as:

$$\begin{aligned}T_i(n)=Z_1(n)\cdot Y_0(n)+Z_2(n)\cdot Y_1(n)+\\ Z_3(n)\cdot Y_2(n)+Z_4(n)\cdot Y_3(n)\end{aligned} \tag{20}$$

and the time domain multi-carrier symbol transmitted is given by:

$$t_i(n)=2\,\mathrm{Re}[IFFT(T_i(n))] \tag{21}$$

In the example described below, only two phasor transformations $Z_1(n)$ and $Z_2(n)$ are used to perform the transformations:

$$\begin{cases} Z_1(n) = e^{j\frac{\pi}{4}n} \dfrac{1-j}{\sqrt{2}} & \text{for } n \text{ even} \\ Z_1(n) = e^{j\frac{\pi}{4}n} \dfrac{1+j}{\sqrt{2}} & \text{for } n \text{ odd} \end{cases} \tag{22}$$

and:

$$\begin{cases} Z_2(n) = e^{j\frac{3\pi}{4}n} \dfrac{1-j}{\sqrt{2}} & \text{for } n \text{ even} \\ Z_2(n) = e^{j\frac{3\pi}{4}n} \dfrac{1+j}{\sqrt{2}} & \text{for } n \text{ odd} \end{cases} \tag{23}$$

The seven transformations applied by the transformation unit TRANSF on the basis of the phasor transformations $Z_1(n)$ and $Z_2(n)$, and corresponding inverse transformations applied by the inverse transformation unit INV TRANSF are listed hereafter.

I. According to the first transformation, the real and imaginary parts of the four partitions $y_0(n)$, $y_1(n)$, $y_2(n)$ and $y_3(n)$ are combined by the transformation unit TRANSF into the following transformed time domain multi-carrier symbol:

$$t_1(n) = 2(a_0(n) + a_3(n)) + \sqrt{2}\left(a_2\left(n+\frac{N}{8}\right) + b_2\left(n+\frac{N}{8}\right) + a_1\left(n+\frac{N}{8}\right) - b_1\left(n+\frac{3N}{8}\right)\right) \tag{24}$$

If the corresponding transformed frequency domain multi-carrier symbol, i.e. the multi-carrier symbol obtained at the output of the fourier transformer FFT in the receiver RX when $t_1(n)$ is supplied thereto, is denoted by $T_1(n)$, the original frequency domain multi-carrier symbol can be recovered from $T_1(n)$ by the inverse transformation unit INV TRANSF if the following inverse transformation is applied:

$$X(n) = \begin{cases} 2T_1(n) & \text{for } n = 0, \frac{N}{2} \\ T_1(n) & \text{for } n \in \{0 \bmod 4, 3 \bmod 4\} \text{and } 0 \le n \le \frac{N}{2}-1 \\ \dfrac{T_1(n)}{Z_1(n)} & \text{for } n \in \{2 \bmod 4\} \text{and } 2 \le n \le \frac{N}{2}-1 \\ \dfrac{T_1(n)}{Z_2(n)} & \text{for } n \in \{1 \bmod 4\} \text{and } 1 \le n \le \frac{N}{2}-1 \\ 0 & \text{for } \frac{N}{2} < n < N \end{cases} \tag{25}$$

Herein, the notation $\{k \bmod l\}$ stands for the set $\{k, k+l, k+2l, k+3l, \ldots\}$, e.g. $\{0 \bmod 4\}$ stands for the set $\{0, 4, 8, 12, \ldots\}$.

II. According to the second transformation, the four partitions $y_0(n)$, $y_1(n)$, $y_2(n)$ and $y_3(n)$ are combined by the transformation unit TRANSF into the following transformed time domain multi-carrier symbol:

$$t_2(n) = 2(a_0(n) + a_1(n)) + \sqrt{2}\left(a_2\left(n+\frac{N}{8}\right) + b_2\left(n+\frac{5N}{8}\right) + a_3\left(n+\frac{3N}{8}\right) + b_3\left(n+\frac{7N}{8}\right)\right) \tag{26}$$

If the corresponding transformed frequency domain multi-carrier symbol is denoted by $T_2(n)$, the original frequency domain multi-carrier symbol can be recovered from $T_2(n)$ by the inverse transformation unit if the following inverse transformation is applied:

$$X(n) = \begin{cases} 2T_2(n) & \text{for } n = 0, \frac{N}{2} \\ T_2(n) & \text{for } n \in \{0 \bmod 4, 1 \bmod 4\} \text{and } 0 \le n \le \frac{N}{2}-1 \\ \dfrac{T_2(n)}{Z_1(n)} & \text{for } n \in \{2 \bmod 4\} \text{and } 2 \le n \le \frac{N}{2}-1 \\ \dfrac{T_2(n)}{Z_2(n)} & \text{for } n \in \{3 \bmod 4\} \text{and } 3 \le n \le \frac{N}{2}-1 \\ 0 & \text{for } \frac{N}{2} < n < N \end{cases} \tag{27}$$

III. In the third transformation, the four partitions $y_0(n)$, $y_1(n)$, $y_2(n)$ and $y_3(n)$ are combined by the transformation unit TRANSF into the following transformed time domain multi-carrier symbol:

$$t_3(n) = 2a_e(n) + \sqrt{2}\left(a_1\left(n+\frac{N}{8}\right) + b_1\left(n+\frac{5N}{8}\right) + a_3\left(n+\frac{3N}{8}\right) + b_3\left(n+\frac{7N}{8}\right)\right) \tag{28}$$

If the corresponding transformed frequency symbol is denoted by $T_3(n)$, the original frequency domain multi-carrier symbol can be recovered from $T_3(n)$ by the inverse transformation unit if the following inverse transformation is applied:

$$X(n)=\begin{cases}2T_3(n) & \text{for } n=0,\frac{N}{2}\\ T_3(n) & \text{for } n \text{ even}\\ \frac{T_3(n)}{Z_1(n)} & \text{for } n\in\{1 \bmod 4\} \text{and } 1\le n\le \frac{N}{2}-1\\ \frac{T_3(n)}{Z_2(n)} & \text{for } n\in\{3 \bmod 4\} \text{and } 3\le n\le \frac{N}{2}-1\\ 0 & \text{for } \frac{N}{2}<n<N\end{cases} \quad (29)$$

IV. According to the fourth transformation, the four partitions $y_0(n), y_1(n), y_2(n)$ and $y_3(n)$ are combined into the following transformed time domain multi-carrier symbol:

$$t_4(n)=2(a_0(n)+a_3(n))+\sqrt{2}\left(a_2\left(n+\frac{3N}{8}\right)+b_2\left(n+\frac{7N}{8}\right)+a_1\left(n+\frac{N}{8}\right)+b_1\left(n+\frac{5N}{8}\right)\right) \quad (30)$$

At the receiver, the original frequency domain multi-carrier symbol can be recovered from the corresponding transformed frequency multi-carrier symbol if the following inverse transformation is applied:

$$X(n)=\begin{cases}2T_4(n) & \text{for } n=0,\frac{N}{2}\\ T_4(n) & \text{for } n\in\{0 \bmod 4, 3 \bmod 4\} \text{and } 0\le n\le \frac{N}{2}-1\\ \frac{T_4(n)}{Z_2(n)} & \text{for } n\in\{2 \bmod 4\} \text{and } 2\le n\le \frac{N}{2}-1\\ \frac{T_4(n)}{Z_1(n)} & \text{for } n\in\{1 \bmod 4\} \text{and } 1\le n\le \frac{N}{2}-1\\ 0 & \text{for } \frac{N}{2}<n<N\end{cases} \quad (31)$$

V. In the fifth transformation, the four partitions $y_0(n), y_1(n), y_2(n)$ and $y_3(n)$ are combined into the following transformed time domain multi-carrier symbol:

$$t_5(n)=2(a_0(n)+a_1(n))+\sqrt{2}\left(a_2\left(n+\frac{3N}{8}\right)+b_2\left(n+\frac{7N}{8}\right)+a_3\left(n+\frac{N}{8}\right)+b_3\left(n+\frac{5N}{8}\right)\right) \quad (32)$$

The original DMT symbol $X(n)$ can be recovered from the corresponding frequency symbol $T_5(n)$ by applying the following transformation:

$$X(n)=\begin{cases}2T_5(n) & \text{for } n=0,\frac{N}{2}\\ T_5(n) & \text{for } n\in\{0 \bmod 4, 1 \bmod 4\} \text{and } 0\le n\le \frac{N}{2}-1\\ \frac{T_5(n)}{Z_2(n)} & \text{for } n\in\{2 \bmod 4\} \text{and } 2\le n\le \frac{N}{2}-1\\ \frac{T_5(n)}{Z_1(n)} & \text{for } n\in\{3 \bmod 4\} \text{and } 3\le n\le \frac{N}{2}-1\\ 0 & \text{for } \frac{N}{2}<n<N\end{cases} \quad (33)$$

VI. In the sixth transformation, the four partitions $y_0(n), y_1(n), y_2(n)$ and $y_3(n)$ are combined into the following transformed time domain multi-carrier symbol:

$$t_6(n)=2a_e(n)+\sqrt{2}\left(a_1\left(n+\frac{3N}{8}\right)+b_1\left(n+\frac{7N}{8}\right)+a_3\left(n+\frac{N}{8}\right)+b_3\left(n+\frac{5N}{8}\right)\right) \quad (34)$$

The original DMT symbol $X(n)$ can be recovered from the corresponding frequency symbol $T_6(n)$ by applying the following transformation:

$$X(n)=\begin{cases}2T_6(n) & \text{for } n=0,\frac{N}{2}\\ T_6(n) & \text{for } n \text{ even}\\ \frac{T_6(n)}{Z_2(n)} & \text{for } n\in\{1 \bmod 4\} \text{and } 1\le n\le \frac{N}{2}-1\\ \frac{T_6(n)}{Z_1(n)} & \text{for } n\in\{3 \bmod 4\} \text{and } 3\le n\le \frac{N}{2}-1\\ 0 & \text{for } \frac{N}{2}<n<N\end{cases} \quad (35)$$

VII. The seventh transformation is obtained by a simple modification of the previous transformation, the corresponding time domain multi-carrier symbol is given by:

$$t_7(n)=2a_e(n)+\sqrt{2}\left(-a_1\left(n+\frac{3N}{8}\right)-b_1\left(n+\frac{7N}{8}\right)+a_3\left(n+\frac{N}{8}\right)+b_3\left(n+\frac{5N}{8}\right)\right) \quad (36)$$

The original frequency domain DMT symbol $X(n)$ can be recovered from the output of the fourier transformer FFT in the receiver, denoted $T_7(n)$, corresponding to the input $t_7(n)$ if the following transformation is applied:

$$X(n)=\begin{cases} 2T_7(n) & \text{for } n=0,\frac{N}{2} \\ T_7(n) & \text{for } n \text{ even} \\ \frac{-T_7(n)}{Z_2(n)} & \text{for } n\in\{1 \bmod 4\} \text{and } 1\le n\le \frac{N}{2}-1 \\ \frac{T_7(n)}{Z_1(n)} & \text{for } n\in\{3 \bmod 4\} \text{and } 3\le n\le \frac{N}{2}-1 \\ 0 & \text{for } \frac{N}{2}<n<N \end{cases} \quad (37)$$

3. Complexity and gain

At the transmitter, one needs to compute a preliminary output to detect if the algorithm must be applied or not, this extra information is encoded into a pilot and send to the receiver. If the proposed algorithm was applied, the complexity at the transmitter measured in terms of extra number of additions and multiplications is limited by (5+3*3+2*3)N real additions for T_1, T_2, T_4 and T_5 and (5+3*4+2*2)N real additions for T_3, T_6 and T_7 and 2N real multiplications for each transformation. At the receiver, the original DMT symbol can be recovered by performing maximum $N/4$ complex multiplications at the output of FFT transformer.

One can mention that the seven transformations described above are an example corresponding to the most complex case. There is still room to improve this technique by decreasing the implementation complexity as will be shown in the next section.

Another remark is that the proposed algorithm is not restricted to ADSL systems. It can be used for all systems wherein a multi-carrier modulation is applied with QAM-constellations of different size used in each carrier.

Figure 1 shows the clip probability versus the allowed PAR for an ADSL implementation using N=512 and 16-QAM in each carrier. The curves denoted by R4 and R8 correspond to Orckit method [7] when applying the original IFFTs and three or seven transformations respectively. Similarly, the curves denoted by S4 and S8 correspond to applying the original IFFTs and transformations I, II, III or the complete transformations respectively. The maximum gain in PAR introduced by the proposed technique is about 3.7 dB instead of 2.7 dB given in [7].

The simulation results show that a given maximum clipping probability can be satisfied whose power level is at least 1.5 dB below the clipping power level obtained by the technique described in [7] corresponding to the *same maximum clipping probability.*

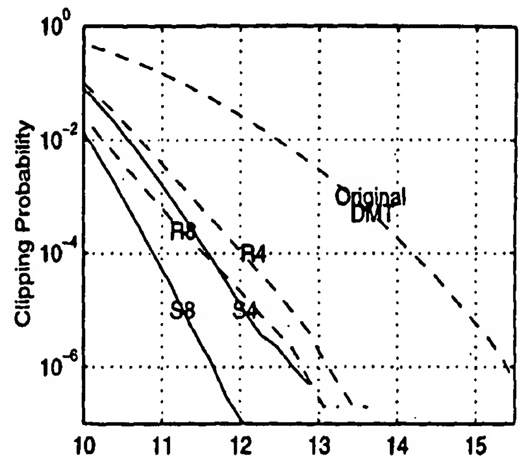

Figure 1. Clipping probability

4. Reducing the implementation complexity

In this section we propose some modifications to reduce the complexity of the previous algorithm. As can be observed, in each transformation described above, there are two scale factors 2 and $\sqrt{2}$, in order to eliminate the factor $\sqrt{2}$, two possible solutions can be proposed:

The first one is to choose a suitable phasor transformation $Z_1(n)$ and $Z_2(n)$ as for example:

$$Z_1(n)=\begin{cases} e^{j\frac{\pi}{2}n} & \text{for } n \text{ even} \\ j.e^{j\frac{\pi}{2}n} & \text{for } n \text{ odd} \end{cases} \quad (38)$$

and:

$$Z_2(n)=\begin{cases} j.e^{j\frac{\pi}{2}n} & \text{for } n \text{ even} \\ e^{j\frac{\pi}{2}n} & \text{for } n \text{ odd} \end{cases} \quad (39)$$

In this case, the number of multiplications at the transmitter will be reduced by 50% at the expense of an increase of the number of additions by an amount 2N per transformation.

The second one is to rotate all the four partitions $Y_0(n)$, $Y_1(n)$, $Y_2(n)$ and $Y_3(n)$ by a specific phasor transformation $Z_i(n)$ as defined in Eq. (18) and then a new frequency sequence can be given by:

$$T_i(n) = Z_a(n)Y_0(n) + Z_b(n)Y_1(n) + Z_c(n)Y_2(n) \quad Z_d(n)Y_3(n) \tag{40}$$

The complexity of this solution is exactly the same as in the previous case with a difference that the entire symbol now is scaled by a factor $\sqrt{2}$ instead of 2.

The simulation results show that the improvements in PAR reduction given by this solution are almost the same given in figure 1.

5. Conclusion

A new method for clip probability reduction is proposed by reducing the PAR of multi-carrier signal. This method is based on the shift property of an FFT in the time domain. The new time domain signals can be easily computed at the transmitter and the receiver can efficiently recover the original frequency symbol from the received frequency symbol. A gain of more than 3.7 dB can be achieved with low complexity and redundancy.

6. References

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